Problem 1.

(i) Find the slope of the tangent of the graph of

$$2x^2 + xy + y^2 = 4$$

at the point $(1, -2)$.

(ii) Find an equation for the tangent line of the graph of

$$9(x^2 - y^2) = x^4$$

at the point $(1, \frac{2}{3}\sqrt{2})$.

(iii) Find the second derivative wrt. x, that is $\frac{d^2y}{dx^2}$ of

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1.$$

Problem 2.

(i) A pyramid has a square base and height 8 meters. The sides of the base increase in length as a constant rate of 3 $\frac{\mathrm{m}}{\mathrm{s}}$. When the sides of the base have length 2 m, at what rate does the volume of the pyramid change? *Note:* The volume of a pyramid is given by the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

(ii) A circular cylinder has length 0.5 m. Its volume changes at a constant rate of 2 $\frac{\mathrm{m}^3}{\mathrm{s}}$. At what rate does the radius change when the radius is 2 m? *Note:* The volume of a circular cylinder is given by the formula $V = \pi r^2 l$, where l is length of the cylinder.

Problem 3.

(i) Find and classify all the relative (local) extrema of the function

$$f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12.$$

Hint: $f'(\frac{1}{2}) = 0$. Factor out $x - \frac{1}{2}$. Use quadratic formula to find the last two.

(ii) Find and classify all the relative (local) extrema of the function

$$f(x) = e^x(x^3 - x^2 + x - 3).$$

Problem 4.

(i) Find the absolute (global) maximum and minimum *values* of the function

$$f(x) = x^3 + x^2 - x - 1$$

on the interval $[-1, \frac{1}{2}]$.

(ii) Find the absolute (global) maximum and minimum *values* of the function

$$f(x) = \sin^2 x + \cos x$$

on the interval $[0, \pi]$.

Problem 5.

(i) Find where the graph of the function

$$f(x) = x^3 - 3x^2 + 2x$$

is concave upward and concave downward, and find all points of infection.

(ii) Find where the graph of the function

$$f(x) = e^{x(2-x)}$$

is concave upward and concave downward, and find all points of infection.

Problem 6. Find the limit. If the limit is infinite, denote it with $\pm\infty$ accordingly.

(i) $\lim_{x\to\infty} \dfrac{x^2 - x + 1}{3x^3 - 5}$

(ii) $\lim_{x\to\infty} \dfrac{2x^3 - x + 15}{90x^2 + 100x + 3000}$

(iii) $\lim_{x\to-\infty} \dfrac{5x^7 + 4x^5 - 2x^3 + 2x}{10x^7 - 20x^3 + \pi}$

(iv) $\lim_{x\to\infty} \dfrac{e^{-x}(3x^2 - 1)}{x^2}$

(v) $\lim_{x\to\infty} \dfrac{1000}{1 + 999e^{-x}}$

(vi) $\lim_{x\to-\infty} \dfrac{1000}{1 + 999e^{-x}}$

Problem 7.

(i) A touch screen whose surface area must be 160 cm^2 is to be placed in a panel with the following restrictions: There must be 1 cm space to each side of the screen, 2 cm space at the top and 3 cm space at the bottom. Find the dimensions of the touch screen that minimize the area of the panel.

(ii) A right triangle has a perimeter of 2 feet. Find the side lengths that maximize the area of the triangle.