Problem 1. Use the definition of the derivative as the limit of a difference qoutient to compute the derivative of the functions below

(i) $f(x) = \sqrt{x}$

(ii) $f(x) = \dfrac{1}{x}$

(iii) $f(x) = 3x^2 + 2x + 1$

Problem 2. Find the derivative of $f(x)$

(i) $f(x) = 42$

(ii) $f(x) = x$

(iii) $f(x) = x^{42}$

(iv) $f(x) = \sqrt{x}$

(v) $f(x) = \dfrac{1}{x^3}$

(vi) $f(x) = \dfrac{1}{\sqrt[5]{x^4}}$

(vii) $f(x) = 42x$

(viii) $f(x) = 21x^2$

(ix) $f(x) = x^4 - 2x^3 + 5x - 7$

(x) $f(x) = \dfrac{3}{2}\sqrt[3]{x^2} - \dfrac{4}{3\sqrt[4]{x^3}} + \dfrac{5}{2}\sqrt[5]{x^2}$

Problem 3. Find the derivative of $f(x)$

(i) $f(x) = \sin x + \cos x$

(ii) $f(x) = 2\sin x - 3\cos x$

(iii) $f(x) = -5e^x$

(iv) $f(x) = 3e^x + 2\sin x$

(v) $f(x) = 2e^x - 3\cos x + 5\sin x - 6e^7$

Problem 4.

(i) Standing on the roof of a 72 feet tall building, a golfer takes a whack at a ball sending it over the edge. The initial velocity of the ball is 24 feet per second and the acceleration due to gravity is -32 feet per second per second.

(a) Determine the position function $s(t)$, that is the height of the ball above the ground as a function of time in seconds.

(b) Determine the velocity function.

(c) What is the velocity of the ball when it hits the ground?

(ii) My father-in-law from Texas always responds to the announcement of a smoked brisket in the appropriate way: shouting "Yeehaa!" and shooting his .38 revolver straight up into the air.
The height of the bullet, in meters, is given by the position formula

$$s(t) = -4.9t^2 + 183t + 2,$$

where time, t, is measured in seconds.

(a) What is the velocity of the bullet after 10 seconds?

(b) What is the height of the bullet when it has velocity $0\frac{\text{m}}{\text{s}}$

My father-in-law can drink a beer in 11 seconds.

(c) How many beers can he drink before he has to move out of the way of the falling bullet? (He waits until the bullet is 2 m above the ground.)

Problem 5. Use the quotient rule to prove the formulas for the derivatives of the trigonometric functions

(i) $\frac{d}{dx}\cot x = -\csc^2 x$

(ii) $\frac{d}{dx}\sec x = \sec x \tan x$

(iii) $\frac{d}{dx}\csc x = -\csc x \cot x$

Problem 6. Find the derivative of $f(x)$.

(i) $f(x) = x \sin x$

(ii) $f(x) = \sin x \cos x$

(iii) $f(x) = 3e^x \cos x$

(iv) $f(x) = x^2 \sin x - \sqrt{x}e^x$

(v) $f(x) = x \csc x$

(vi) $f(x) = \dfrac{x^3}{2x^4 - 5}$

(vii) $f(x) = \dfrac{x^2 + 2}{2x^3 - 5x^2 - 3x + 7}$

(viii) $f(x) = \dfrac{\sin x - \cos x}{e^x + x}$

Problem 7. Find the derivative of $f(x)$.

(i) $f(x) = \sin(2x)$

(ii) $f(x) = \sqrt{2x^3 - x^2 + 1}$

(iii) $f(x) = \cos(x^2)$

(iv) $f(x) = e^{\sqrt{x}}$

(v) $f(x) = \ln(x^2 - 3x + 2)$

(vi) $f(x) = \sqrt{x + \sqrt{x}}$

(vii) $f(x) = \ln\left(\dfrac{\sqrt{2x^2 - 3x + 2}}{(x^2 + 1)^2}\right)$