Problem 1. Find the limit, given it exists. If the limit doesn't exist, state why.

(i) $\lim_{x\to 2} \dfrac{7x}{x^2-x-2}$

(ii) $\lim_{x\to 1} \dfrac{x^3-x^2+x-1}{x+2}$

(iii) $\lim_{x\to 0} \dfrac{|x|}{x}$

(iv) $\lim_{x\to 1} \sin\left(\dfrac{1}{x-1}\right)$

(v) $\lim_{x\to \frac{\pi}{2}} \tan x$

(vi) $\lim_{x\to -1} f(x)$,
where $f(x) = \begin{cases} x+1, & \text{if } x \neq -1 \\ 42, & \text{if } x = -1 \end{cases}$

(vii) $\lim_{x\to 3} \dfrac{2x^2-5x-3}{x-3}$

(viii) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{2x-3}-1}$

(ix) $\lim_{x\to 0} \dfrac{\sin(5x)}{3x}$

(x) $\lim_{x\to 0} \dfrac{\sin(2x)e^x}{3x\cos x}$

(xi) $\lim_{x\to 0} x\cot x$

(xii) $\lim_{x\to -1} f(x)$, where
$f(x) = \begin{cases} x^2\cos\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

Problem 2. Find the discontinuities (if any) for each of the functions below. Determine whether they are removable, and, in that case, (re)define the function value to make it continuous at that point.

(i) $f(x) = \dfrac{x^2-x}{2x}$

(ii) $f(x) = \dfrac{x-4}{\sqrt{x-3}-1}$

(iii) $f(x) = \dfrac{2}{x^2-1}$

(iv) $f(x) = \dfrac{x+1}{|x-1|}$

(v) $f(x) = \dfrac{x}{\sin x}$

Problem 3. For each of the functions below, determine the value k for which the function is everywhere continuous. Justify your answer.

(i) $f(x) = \begin{cases} \frac{x^2+4x+3}{x+1}, & \text{if } x \neq -1 \\ k, & \text{if } x = -1 \end{cases}$

(ii) $f(x) = \begin{cases} \cos x + 4, & \text{if } x \leq 0 \\ e^x + k, & \text{if } x > 0 \end{cases}$

Problem 4. Find all the vertical asymptotes (if any) of the functions below.

(i) $f(x) = \dfrac{1}{x^2-2x+1}$

(ii) $f(x) = \dfrac{2x^2+x-1}{x-1}$

(iii) $f(x) = \dfrac{1}{1-x}$

(iv) $f(x) = \dfrac{x+1}{x^2-1}$

(v) $f(x) = \sec x$

Problem 5. Find the one-sided limit, given it exists (indicate unboundedness by $\pm\infty$).

(i) $\lim_{x\to 1^-} \dfrac{1}{1-x}$, $\lim_{x\to 1^+} \dfrac{1}{1-x}$

(ii) $\lim_{x\to 1^-} \dfrac{x-1}{|x-1|}$, $\lim_{x\to 1^+} \dfrac{x-1}{|x-1|}$

(iii) $\lim_{x\to \frac{\pi}{2}^-} \sec x$, $\lim_{x\to \frac{\pi}{2}^+} \sec x$

(iv) $\lim_{x\to 2^-} \dfrac{2x-4}{|x-2|}$, $\lim_{x\to 2^+} \dfrac{2x-4}{|x-2|}$

(v) $\lim_{x\to 2^-} 2[\![x]\!]+1$, $\lim_{x\to 2^+} 2[\![x]\!]+1$

(vi) $\lim_{x\to 2^-} \dfrac{x^2-x-2}{|\frac{1}{2}x-1|}$, $\lim_{x\to 2^+} \dfrac{x^2-x-2}{|\frac{1}{2}x-1|}$